

# EFFECT OF VISCOUS DISSIPATION TERM IN ENERGY EQUATION ON MHD FREE CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND HEAT SOURCE

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## ABSTRACT

The theoretically examines an unsteady free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate is analyzed by taking into an account the heat due to viscous dissipation under the influence of a uniform transverse magnetic field. The problem is governed by coupled nonlinear partial differential equations. Dimensionless equations of the problem have been solved by using perturbation technique. The effects of various parameter involving in the equations are discussed on velocity and temperature fields through graphs. Skin friction coefficient is derived, discussed numerically and presented in tabular form.

**Keywords:** MHD, free convection, viscous dissipation, exponentially accelerated plate, variable temperature and heat source

## INTRODUCTION

Deformation and flow of materials require energy. This mechanical energy is dissipated, i.e. during the flow it is converted into internal energy (heat) of the material. This phenomenon can be demonstrated by performing a simple experiment with a metal paper clip: bend the clip wide open and close it repeatedly until the clip breaks. Now touch the metal near the region of break and feel the high temperature. The mechanical energy for bending the metal has been converted into internal energy. The increase of internal energy expresses itself in a temperature rise. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flows such as injection molding or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed

aircraft raises the temperature of the skin. In completely different application, the dissipation function is used to define the viscosity of dilute suspensions. Viscous dissipation for a fluid with suspended particles is equated to the viscous dissipation in a pure Newtonian fluid, both being in the same flow (same macroscopic velocity gradient).

Hydro magnetic flows and heat transfer have become more important in recent years because of its varied applications in agricultural engineering and petroleum industries. Recently, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer such as metallurgical processing. Melt refining involves magnetic field applications to control excessive heat transfer rate. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems etc. In view of above applications some of related authors have been depicted. Many authors studied of the above mention applications, has been studied by Ch Kesavaiah et. al. [1] has been considered the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction, Satyanarayana et. al. [2] examine on viscous dissipation and thermal radiation effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving porous plate, Ch Kesavaiah et. al. [3] motivated study on radiation and mass transfer effects on moving vertical plate with variable temperature and viscous dissipation, Chenna Kesavaiah et. al. [4] explained in detailed on natural convection heat transfer oscillatory flow of an elastico-viscous fluid from vertical plate, Chenna Kesavaiah and Satyanarayana [5] reviewed on MHD and Diffusion Thermo effects on flow accelerated vertical plate with chemical reaction, Chenna Kesavaiah et. al. [6] studied the radiation and Thermo - Diffusion effects on mixed convective heat and mass transfer flow of a viscous dissipated fluid over a vertical surface in the presence of chemical reaction with heat source, Karunakar Reddy et. al. [7] observed that the MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction, Ch Kesavaiah et. al. [8] represented on effects of radiation and free convection currents on unsteady Couette flow between two vertical parallel plates with constant heat flux and heat source through porous medium.

Deformation and flow of materials requires energy. This mechanical energy is dissipation, i.e. during the flow it is converted into internal energy (heat) of the material. This phenomenon can be demonstrated by performing a simple experiment with a metal paper clip: bend the clip

wide open and close it repeatedly until the clip breaks. Now, touch the metal near the region of break and feel the high temperature. The mechanical energy for bending the metal has been converted into internal energy. The increase of internal energy expresses itself in a temperature rise. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flow such as injection molding or extrusion at high rates. Aerodynamic heating in thin boundary layer around high speed aircraft raises the temperature of the skin. In a completely different application, the dissipation function is used to define the viscosity of dilute suspensions. Viscous dissipation for a fluid with suspended particles is equated to the viscous dissipation in a pure Newtonian fluid, both being in the same flow (same macroscopic velocity gradient). In view of the above authors considered. Satya Narayana et. al. [9] illustrated on the effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system, Rajaiah et. al. [10] has been studied an unsteady MHD free convective fluid flow past a vertical porous plate with Ohmic heating In the presence of suction or injection, Chenna Kesavaiah and Sudhakaraiah [11] expressed the effects of heat and mass flux to MHD flow in vertical surface with radiation absorption, Rajaiah Sudhakaraiah [12] explained in detailed on an unsteady MHD free convection flow past an accelerated vertical plate with chemical reaction and Ohmic heating, Haranth and Sudhakaraiah [13] motivated study on viscosity and Soret effects on unsteady hydromagnetic gas flow along an inclined plane Rajaiah and Sudhakaraiah [14] observed that the radiation and Soret effect on Unsteady MHD flow past a parabolic started vertical plate in the presence of chemical reaction with magnetic dissipation through a porous medium, Rajaiah et. al. [15] motivated study on chemical and Soret effect on MHD free convective flow past an accelerated vertical plate in presence of inclined magnetic field through porous medium.

Mostly fluids which are very useful in our daily life and industry do not obey the Newtonian expression of viscosity, for examples paints, oil, lubricating greases, human blood, honey, biological fluids etc. These fluids are called non-Newtonian. Due to their importance in our daily life, in last few years many studies have been reported in which characteristics of non-Newtonian fluids are explored. Due to the application of viscous dissipation, wall conduction, boundary plate thickness in lubrication industries, cooling of nuclear reactors, cooling of electric appliances etc., it is very important to investigate such fluid properties in natural convection flow through a vertical channel. Results from this investigation will assist designers to improve on the performance of mechanical systems involving viscous dissipation and heat transfer in channels or annulus as it is in combustion in auto piston. In view of the above some of the authors were Ramesh Babu et. al. [16] expressed their ideas on radiation

effect on MHD free convective heat absorbing Newtonian fluid with variable temperature, Chenna Kesavaiah et. al. [17] noticed that the heat and mass transfer effects over isothermal infinite vertical plate of Newtonian fluid with chemical reaction, Chenna Kesavaiah et. al. [18] observed the influence of joule heating and mass transfer effects on MHD mixed convection flow of chemically reacting fluid on a vertical surface, Bal Reddy et. al. [19] explained in detailed on a note on heat transfer of MHD Jeffrey fluid over a stretching vertical surface through porous plate, Chenna Kesavaiah et. al. [20] expressed the radiation and mass transfer effects on MHD mixed convective flow from a vertical surface with heat source and chemical reaction, Chenna Kesavaiah et. al. [21] declared that he radiation, radiation absorption, chemical reaction and hall effects on unsteady flow past an isothermal vertical plate in a rotating fluid with variable mass diffusion with heat source, Chenna Kesavaiah et. al. [22] states that the chemical reaction, heat and mass transfer effects on MHD peristaltic transport in a vertical channel through space porosity and wall properties, Chenna Kesavaiah distinguished the MHD effect on boundary layer flow of an unsteady incompressible micropolar fluid over a stretching surface, Chenna Kesavaiah et. al. [23] discovered the chemical reaction and MHD effects on free convection flow of a viscoelastic dusty gas through a semi infinite plate moving with radiative heat transfer, Chenna Kesavaiah et. al. [24] states that the radiative MHD Walter's Liquid-B Flow past a semi-infinite vertical plate in the presence of viscous dissipation with a heat source, Chenna Kesavaiah et. al. [25] illustrated MHD effect on convective flow of dusty viscous fluid with fraction in a porous medium and heat generation, Rami Reddy et. al. [26] demonstrated on Hall effect on MHD flow of a visco-elastic fluid through porous medium over an infinite vertical porous plate with heat source, Chenna Kesavaiah et. al. [27] has been studied chemical reaction and radiation absorption effects on convective flows past a porous vertical wavy channel with travelling thermal waves, Chenna Kesavaiah et. al. [28] represented on radiation effect to MHD oscillatory flow in a channel filled through a porous medium with heat generation, Mallikarjuna Reddy et. al. [29] worked out on effects of radiation and thermal diffusion on MHD heat transfer flow of a dusty viscoelastic fluid between two moving parallel plates.

The object of the present paper is to study the transient free convection flow of an incompressible dissipative viscous fluid past an exponentially accelerated vertical plate on taking into account viscous dissipative heat, under the influence of a uniform transverse magnetic field in the presence of variable surface temperature with heat source. The dimensionless governing equations are solved by using perturbation technique.

## **MATHEMATICAL FORMULATION**

Here the unsteady hydro magnetic free convective flow of viscous incompressible fluid past an exponentially accelerated infinite vertical plate with variable temperature is considered. The  $x'$  – axis is taken along the plate in the vertically upward direction and the  $y'$  – axis is taken normal to the plate. At time  $t' \leq 0$ , the plate and fluid are at the same temperature  $T_\infty$ . At  $t' > 0$ , the plate is exponentially accelerated with a velocity  $u' = u_0 \exp(at')$  in its own plane and the plate temperature is raised linearly with time  $t'$  under these conditions, the flow variables are functions of  $t'$  and  $y'$  alone. A uniform magnetic field of intensity  $H_0$  is applied in the  $y'$  – direction. Therefore the velocity and the magnetic field are given by  $\bar{q} = (u, 0)$  and  $\bar{H} = (0, H_0)$ . The fluid being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in comparison with the applied magnetic field in the absence of any input electric field. Then by usual Boussinesq's and boundary layer approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T_\infty) + \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma\mu_e H_0^2}{\rho} u' \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \mu \left( \frac{\partial u'}{\partial y'} \right)^2 + Q'(T' - T_\infty) \quad (2)$$

With the following initial and boundary conditions

$$\begin{aligned} t' \leq 0; u' = 0, T' = T_\infty & \quad \text{for all } y' \\ t' \geq 0; u' = u_0 \exp(at'), T' = T_\infty + (T_w - T_\infty) & \quad \text{as } y' = 0 \\ u' \rightarrow 0, T' \rightarrow T_\infty & \quad \text{as } y' \rightarrow \infty \end{aligned} \quad (3)$$

On introducing the following non-dimensional quantities (4) in equations (1) and (2) reduces to

$$\begin{aligned} u = \frac{u'}{u_0}, y = \frac{u_0 y'}{\nu}, t = \frac{t' u_0^2}{\nu}, \theta = \frac{T' - T_\infty}{T_w - T_\infty}, M = \frac{\sigma\mu_e H_0^2 \nu}{\rho u_0^2}, \text{Pr} = \frac{\mu C_p}{\kappa} \\ Q = \frac{\nu Q'}{\rho C_p u_0^2}, \text{Ec} = \frac{u_0^2}{C_p (T_w - T_\infty)}, \text{Gr} = \frac{\nu \beta g (T_w - T_\infty)}{u_0^3}, a = \frac{a' \nu}{u_0^2} \end{aligned} \quad (4)$$

Where  $u$  is the velocity along the  $x'$  – axis,  $\nu$  is the velocity along  $y'$  – axis,  $\nu$  is the kinematic viscosity,  $g$  is the acceleration due to gravity,  $T$  is the temperature of the fluid,  $\beta$  is the coefficient of volume expansion,  $C_p$  is the specific heat at constant pressure,  $\epsilon$  is a constant,  $\sigma$  is the Stefan-Boltzmann constant,  $\lambda$  is the mean absorption coefficient,  $T_w$  is the

temperature of the surface,  $T_\infty$  is the temperature far away from the surface,  $Pr$  is the Prandtl number,  $Gr$  is the Grashoff number,  $q_r$  is radiative heat flux in  $y$  direction,  $\rho$  is the density,  $t$  is the time,  $k$  is the thermal conductivity of the fluid,  $n$  is the frequency of oscillation of the fluid and  $E$  is the Ecker number,  $B_0$  is uniform magnetic field strength,  $M$  is the magnetic field parameter which is the ratio of magnetic force to the inertial force.

We get the following equations which are dimensionless

$$\frac{\partial u}{\partial t} = Gr\theta + \frac{\partial^2 u}{\partial y^2} - Mu \quad (5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + E \left( \frac{\partial u}{\partial y} \right)^2 + Q\theta \quad (6)$$

The initial and boundary conditions in dimensionless form are as follows

$$\begin{aligned} t' \leq 0; u = 0, \theta = 0 & \quad \text{for all } y \\ t' > 0; u = \exp(at), \theta = t & \quad \text{at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0 & \quad \text{as } y \rightarrow \infty \end{aligned} \quad (7)$$

### SOLUTION OF THE PROBLEM

Equation (5) – (6) are coupled, non – linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (9). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

$$\begin{aligned} u &= u_0(y) + \varepsilon e^{at} u_1(y) \\ \theta &= \theta_0(y) + \varepsilon e^{at} \theta_1(y) \end{aligned} \quad (8)$$

Substituting equation (8) in to equations (6) – (7)

$$u_0'' - Mu_0 = -Gr\theta_0 \quad (9)$$

$$u_1'' + u_1' - (M + a)u_1 = -Gr\theta_1 \quad (10)$$

$$\theta_0'' + QPr\theta_0 = -Pr u_0'^2 \quad (11)$$

$$\theta_1'' + QPr\theta_1 = -2EcPr u_0 u_1 \quad (12)$$

The initial and boundary conditions in dimensionless form are as follows

$$\begin{aligned} u_0 = \exp(at), u_1 = 0, \theta_0 = t, \theta_1 = 0 & \quad \text{at } y = 0 \\ u_0 \rightarrow 0, u_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0 & \quad \text{as } y \rightarrow \infty \end{aligned} \quad (13)$$

The equations (9) - (12) are still coupled and non-linear, whose exact solutions are not possible. So we expand  $u_0, u_1, \theta_0, \theta_1, C_0, C_1$  in terms  $(f_0, f_1)$  of  $Ec \ll 1$  in the following form, as the Eckert number is very small for incompressible flows.

$$\begin{aligned} f_0(y) &= f_{01}(y) + Ec f_{02}(y) \\ f_1(y) &= f_{11}(y) + Ec f_{12}(y) \end{aligned} \quad (14)$$

Substituting (14) in equations (9) - (12), equating the coefficients of  $Ec$  to zero and neglecting the terms in  $Ec^2$  and higher order, we get the following equations

The zeroth order equations are

$$u''_{00} - Mu_0 = -Gr\theta_{00} \quad (15)$$

$$u''_{01} - Mu_{01} = -Gr\theta_{01} \quad (16)$$

$$\theta''_{00} + QPr\theta_{00} = 0 \quad (17)$$

$$\theta''_{01} + QPr\theta_{01} = -Pr u'^2_{00} \quad (18)$$

Boundary conditions are

$$\begin{aligned} u_{00} &= \exp(at), u_{10} = 0, \theta_{00} = t, \theta_{10} = 0 \quad \text{at } y = 0 \\ u_{11} &\rightarrow 0, u_{12} \rightarrow 0, \theta_{11} \rightarrow 0, \theta_{12} \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (19)$$

The first order equations are

$$u''_{11} - (M + a)u_{11} = -Gr\theta_{11} \quad (20)$$

$$u''_{12} + u'_{12} - (M + a)u_{12} = -Gr\theta_{12} \quad (21)$$

$$\theta''_{11} + (Q - a)Pr\theta_{11} = 0 \quad (22)$$

$$\theta''_{12} + (Q - a)Pr\theta_{12} = -2Pr u_{00}u_{11} \quad (23)$$

The boundary conditions are

$$\begin{aligned} u_{00} &= \exp(at), u_{10} = 0, \theta_{00} = t, \theta_{10} = 0 \quad \text{at } y = 0 \\ u_{11} &\rightarrow 0, u_{12} \rightarrow 0, \theta_{11} \rightarrow 0, \theta_{12} \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (24)$$

Solving equations (15) - (18) under the boundary conditions (19) and equations (20) - (23) under the boundary conditions (24), and using Equations (14) and (8), we obtain the velocity, and temperature distributions in the boundary layer as

$$u(y) = A_1 e^{m_2 y} + A_2 e^{m_6 y} + Ec \left\{ A_3 e^{m_6 y} + A_4 e^{2m_4 y} + A_5 e^{2m_2 y} + A_6 e^{(m_2 + m_4) y} + A_7 e^{m_6 y} \right\}$$

$$\theta(y) = t e^{m_2 y} + Ec \left\{ B_1 e^{2m_4 y} + B_2 e^{2m_2 y} + B_3 e^{(m_2 + m_4) y} + B_4 e^{m_6 y} \right\}$$

**Skin friction**

$$\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} = m_2 A_1 + m_6 A_2 + Ec \{ A_3 m_6 + 2A_4 m_4 + 2A_5 m_5 + (m_2 + m_4) A_6 + m_{16} A_7 \}$$

**Rate of heat transfer**

$$Nu = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = t m_2 + Ec \{ 2B_1 m_4 + 2B_2 m_2 + (m_2 + m_4) B_3 + B_4 m_6 \}$$

**RESULTS AND DISCUSSION**

In order to get the physical insight into the problem the velocity, temperature, skin-friction and the rate of heat transfer are shown graphically and some numerical computations are also performed for different values of the physical parameters like Radiation parameter ( $R$ ), Magnetic parameter ( $M$ ), Heat source parameter ( $Q$ ), Time ( $t$ ), exponential index ( $a$ ), porous permeability parameter ( $K$ ), Eckert number ( $Ec$ ) and Prandtl number ( $Pr$ ). The effect of magnetic parameter  $M$  on velocity  $u$  when  $M = 0, 2.0, 4.0$ ,  $Gr = 5.0, Ec = 1.0, Pr = 0.71, a = 0.5$ , and  $t = 0.2 \& 0.6$  is illustrated in **figure (1)**. From **figure (1)** it is observed that velocity  $u$  decreases as the magnetic parameter 'M' increases. It is because of that, the application of transverse magnetic field will result in a resistive type force, known as Lorentz force, which tend to resist the fluid flow and thus reduces velocity. The effect of Grashof number ( $Gr$ ) for heat transfer on the velocity of the flow field when  $Gr = 1.0, 2.0, 3.0, 4.0$ ,  $M = 1.0, Ec = 1.0$ ,  $Pr = 0.71$ ,  $a = 0.5$  and  $t = 0.2 \& 0.6$  is presented in **figure (2)**. In the **figure (2)**, the velocity of the flow field is plotted against 'y' for different values of the respective Grashof numbers keeping other parameters of flow field constant. It is observed that the velocity increases with increasing values of the thermal Grashof number. This is due to enhancement in buoyancy force. The effect of velocity for different values of  $a = 0.5, 0.7, 0.9, 1.1$  and  $M = 1.0$ ,  $Ec = 1.0$ ,  $Pr = 0.71, Gr = 1.0$  at  $t = 0.2 \& 0.6$  are studied and presented in **figure (3)**. It is observed that the velocity increases with increasing values of 'a'. Figure (4) demonstrates the velocity distribution for different values of Eckert number  $Ec$  when  $Gr = 1.0, M = 1.0, Pr = 0.71, a = 0.5$  and  $t = 0.2 \& 0.6$ . It is observed that increasing values of  $Ec$  is to increase the velocity distribution in flow region. This is due to the heat energy stored in the liquid because of the frictional heating. The velocity profiles for different values of time  $t = 0.2, 0.4, 0.6$  and  $a = 0.5, Gr = 1.0$ ,  $Pr = 0.71$ ,  $M = 1.0, Ec = 1.0$  are shown in **figure (5)**. It is observed that the velocity increases with increasing values of time. The velocity profiles for different values of time  $Q = 1.0, 2.0, 3.0, 4.0$  and  $a = 0.5, Gr = 1.0$ ,



$Pr = 0.71$ ,  $M = 1.0$ ,  $Ec = 1.0$  and  $t = 0.2$  &  $0.6$  are shown in **figure (6)**. It is observed that the velocity decreases with increasing values of time. The influence of flow parameters on the temperature  $\theta$  are shown in the **figures (7) to (12)**. The effect of magnetic parameter  $M$  on temperature  $\theta$  when  $M = 0, 2, 4$ ,  $Gr = 5$ ,  $E = 1$ ,  $Pr = 0.71$ ,  $a = 0.5$ , and  $t = 0.2$  &  $0.6$  is illustrated in **figure (7)**. From **figure (7)** it is observed that  $\theta$  increases as 'M' increases in the vicinity of the plate and then decreases in the remaining flow region. The effect of Grashof number ( $Gr$ ) for heat transfer on the temperature when  $Gr = 2, 5, 10$ ,  $M = 1.0$ ,  $Ec = 1.0$ ,  $Pr = 0.71$ ,  $a = 0.5$ , and  $t = 0.2$  &  $0.6$  is presented in **figure (8)**. It is observed that the temperature decreases with increasing values of  $Gr$  in the vicinity of the plate and then increases in the remaining flow region. The effect of temperature for different values of  $a = 0.5, 0.9$  and  $Gr = 1.0$ ,  $M = 1.0$ ,  $Ec = 1.0$ ,  $Pr = 0.71$  at  $t = 0.2$  &  $0.6$  are studied and presented in **figure (9)**. It is observed that temperature increases with increasing values of 'a'. **Figure (10)** depicts the temperature distribution for different values of Eckert number  $Ec$  when  $Gr = 1.0$ ,  $M = 1.0$ ,  $Pr = 0.71$ ,  $a = 0.5$  and  $t = 0.2$  &  $0.6$ . It is observed that the temperature increases with increasing values of  $Ec$ . The temperature profiles for different values of time  $t = 0.2, 0.4, 0.6, 0.8$ .  $Gr = 1.0$ ,  $M = 1.0$ ,  $Ec = 1.0$ ,  $a = 0.5$  are shown in **figure (11)**. It is observed that the temperature increases with increasing values of time. The temperature profiles for different values of time  $Q = 1.0, 2.0, 3.0, 4.0$ ,  $Gr = 1.0$ ,  $M = 1.0$ ,  $Ec = 1.0$ ,  $a = 0.5$  at  $t = 0.2$  &  $0.6$  are shown in **figure (12)**. It is observed that the temperature decreases with increasing values of  $Q$ . The effect of Prandtl number  $Pr$  is important in temperature profiles. The temperature profiles are calculated for different values of time for water ( $Pr = 7.0$ ) and air ( $Pr = 0.71$ ) are demonstrated in **figure (13)**. It is observed that the temperature increases with increasing time 't'. Comparing both the curves of **figure (13)**, also observed that when  $Pr = 7.0$  the temperature increases in the region  $0 \leq y \leq 7.0$  with maximum at  $y = 0.2$  and then decreases in the remaining region. **Figure (14)** depict the effects of the magnetic parameter  $M$ , on the Nusselt number ( $Nu$ ), respectively. It is observed from this figure that as  $M$  increases, the Nusselt number decrease. In order to ascertain the accuracy of the numerical results, the present study is compared with the previous study.

The numerical values of skin friction,  $\tau$  is presented in the table (1). It is observed from the table that an increase in the thermal Grashof number, Eckert number, Prandtl number lead to decrease in the value of the skin-friction, but the trend is just reversed with increasing 'a',  $M$  or 't'.

## CONCLUSIONS

In this paper effects of heat transfer and viscous dissipation on MHD free convection flow past an exponentially accelerated vertical plate with variable surface temperature have been studied numerically. Perturbation technique is employed to solve the equations governing the flow. From the present numerical investigation, following conclusions have been drawn: Velocity increases with increase in the thermal Grashof number ( $Gr > 0$ ), acceleration parameter ' $a$ ', Eckert number  $Ec$  and time ' $t$ '.

- Velocity decreases with an increase in magnetic parameter ( $M$ )
- As  $M$  increases, temperature increases near the plate and decreases in the remaining flow region
- With increasing values of  $Gr$ , temperature decreases near the plate and increases in the remaining flow region
- Temperature increases with increasing values of ' $a$ ', Eckert number  $Ec$  and time ' $t$ '
- Increase in the thermal Grashof number ( $Gr > 0$ ), Eckert number  $Ec$ , Prandtl number  $Pr$  lead to decrease in the value of the skin-friction, but the trend is just reversed with increasing ' $a$ ',  $M$  or ' $t$ '.

**Table (1):** The numerical values of skin friction ( $\tau$ )

| $Gr$ | $a$ | $Ec$ | $M$ | $t$ | $Pr$ | $\tau$ |
|------|-----|------|-----|-----|------|--------|
| 2.0  | 2.0 | 1.0  | 2.0 | 0.2 | 0.71 | 3.0879 |
| 5.0  | 2.0 | 1.0  | 2.0 | 0.2 | 0.71 | 2.9054 |
| 10.0 | 2.0 | 1.0  | 2.0 | 0.2 | 0.71 | 2.5901 |
| 5.0  | 0   | 1.0  | 2.0 | 0.2 | 0.71 | 1.4801 |
| 5.0  | 5.0 | 1.0  | 2.0 | 0.2 | 0.71 | 6.8643 |
| 5.0  | 2.0 | 0    | 2.0 | 0.2 | 0.71 | 3.0535 |
| 5.0  | 2.0 | 0.5  | 2.0 | 0.2 | 0.71 | 2.9824 |
| 5.0  | 2.0 | 1.0  | 4.0 | 0.2 | 0.71 | 3.4904 |
| 5.0  | 2.0 | 1.0  | 6.0 | 0.2 | 0.71 | 4.0010 |
| 5.0  | 2.0 | 1.0  | 2.0 | 0.4 | 0.71 | 3.6975 |
| 5.0  | 2.0 | 1.0  | 2.0 | 0.6 | 0.71 | 5.1056 |
| 5.0  | 2.0 | 1.0  | 2.0 | 0.2 | 7.0  | 2.6589 |

## APPENDIX

$$m_2 = -\sqrt{\beta_1}, m_4 = m_{16} = -\sqrt{M}, m_6 = -\sqrt{Q}, m_8 = -\sqrt{\beta_2}, m_{10} = m_{14} = -\sqrt{\beta_3}$$

$$\beta_1 = (QPr), \beta_2 = \beta_4 = (Q+a)Pr, \beta_3 = (M+a)$$

$$A_1 = -\frac{Gr t}{m_2^2 - M}, A_2 = e^{at} - A_1, A_3 = -\frac{Gr B_4}{4m_6^2 - M}, A_4 = -\frac{Gr B_1}{4m_4^2 - M}, A_5 = -\frac{Gr B_2}{4m_2^2 - M}$$

$$A_6 = -\frac{Gr B_3}{(m_2 + m_4)^2 - M}, A_7 = -(A_3 + A_4 + A_5 + A_6)$$

$$B_1 = -\frac{EcA_2^2}{4m_4^2 - Q}, B_2 = -\frac{EcA_1^2}{4m_2^2 - Q}, B_3 = -\frac{2EcA_1A_2}{(m_2 + m_4)^2 - Q}, B_4 = -(B_1 + B_2 + B_3)$$

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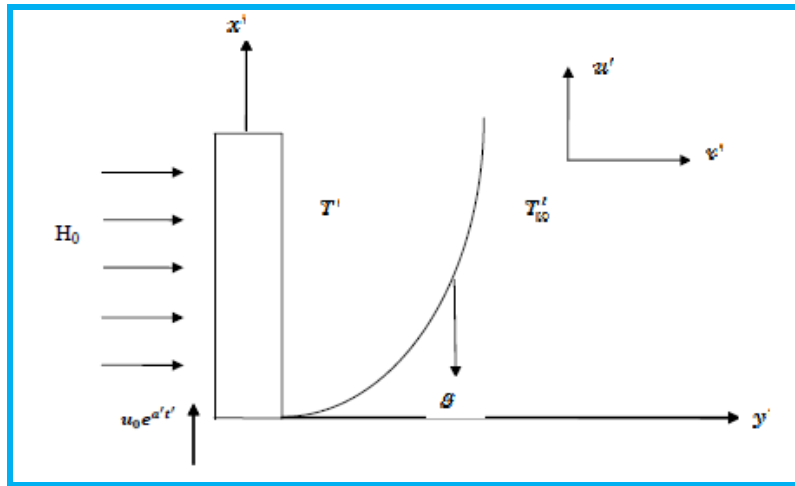


Figure: Flow configuration and coordinate system

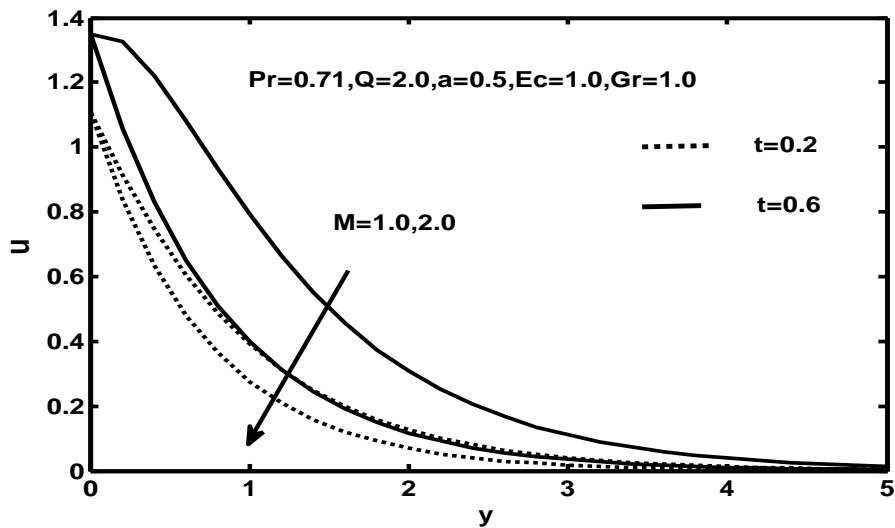


Figure. (1): Velocity profiles for different values of M

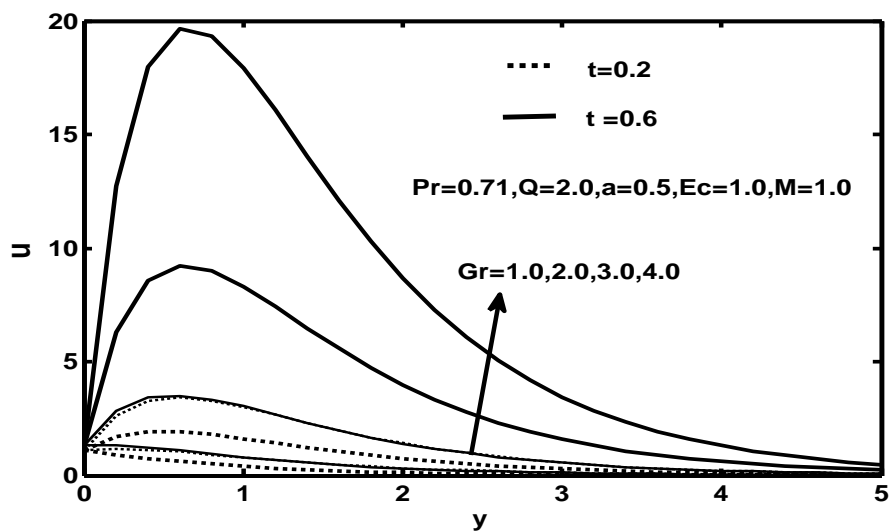


Figure (2): Velocity profile for different values of Gr

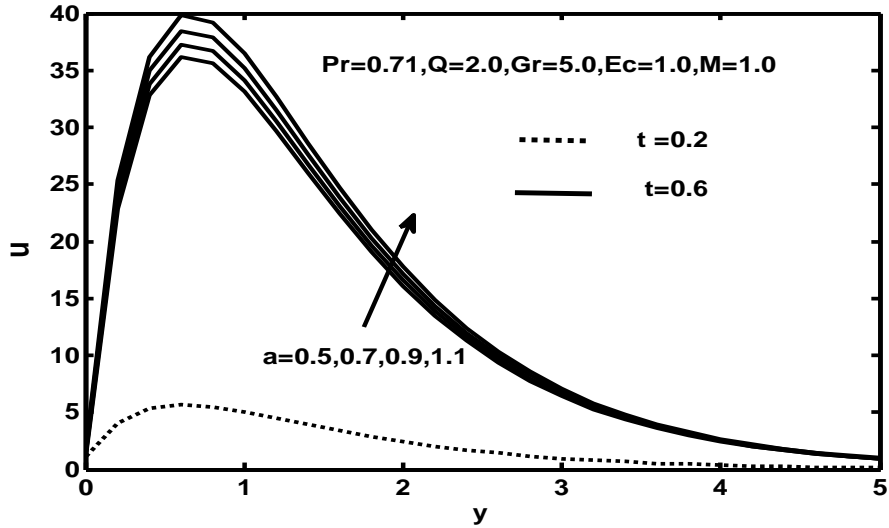


Figure (3): Velocity profile for different values of 'a'

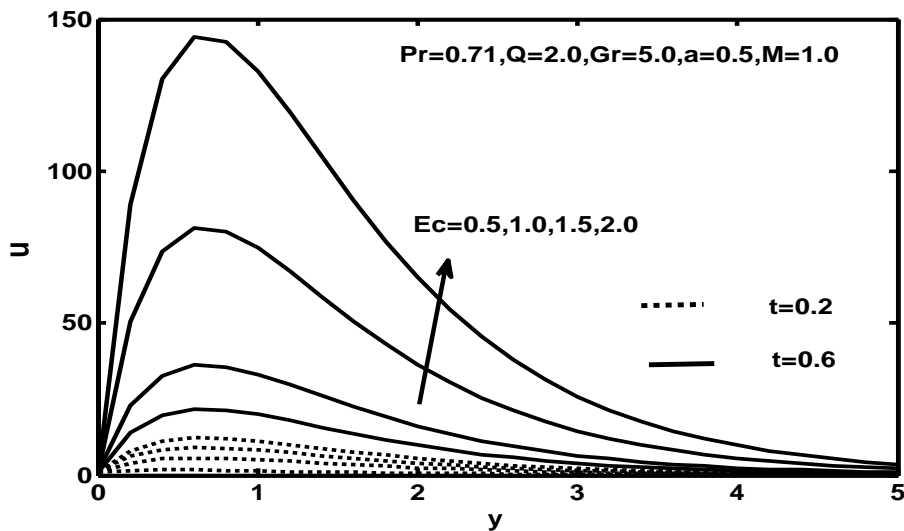


Figure. (4): Velocity profile for different values of E

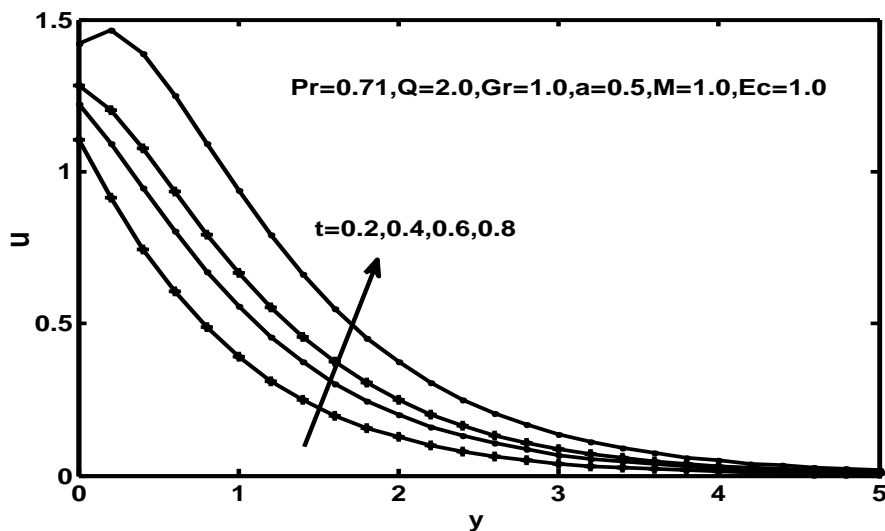


Figure (5): Velocity profile for different values of 't'



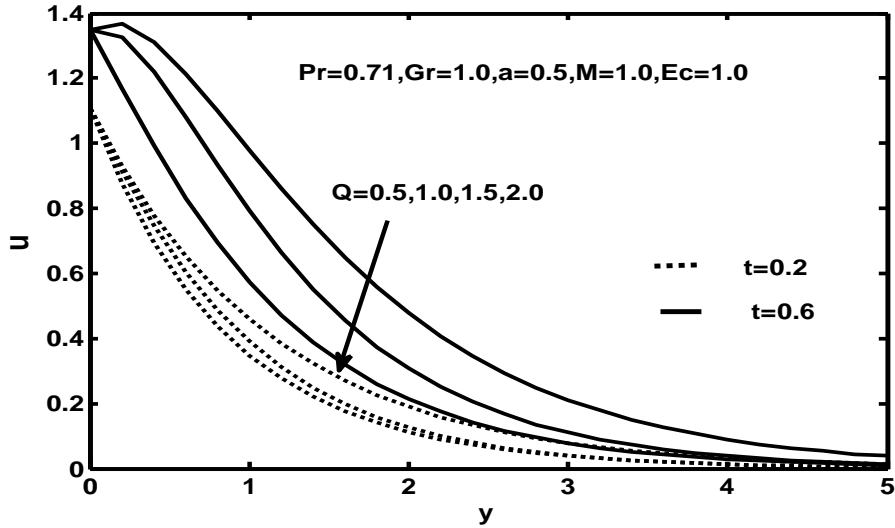


Figure (6): Velocity profile for different values of Q

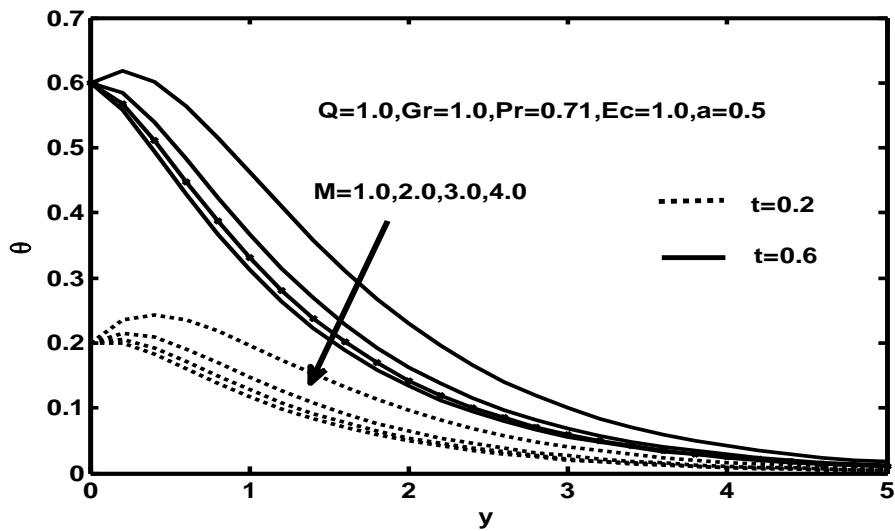


Figure (7): Temperature profile for different values of M

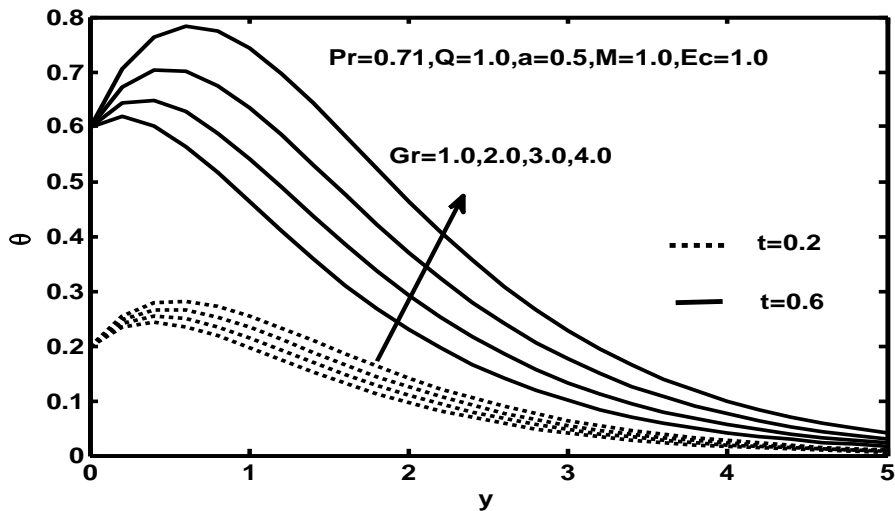


Figure (8): Temperature profile for different values of Gr

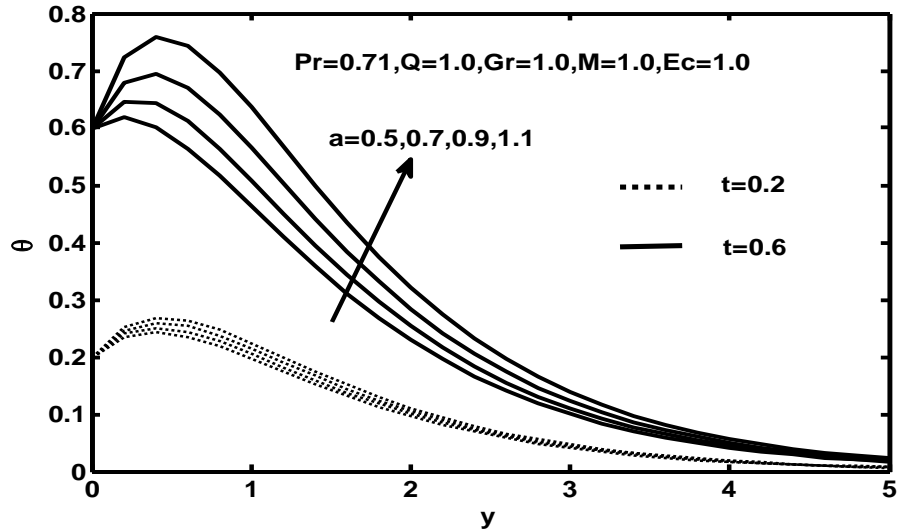


Figure (9): Temperature profile for different values of 'a'

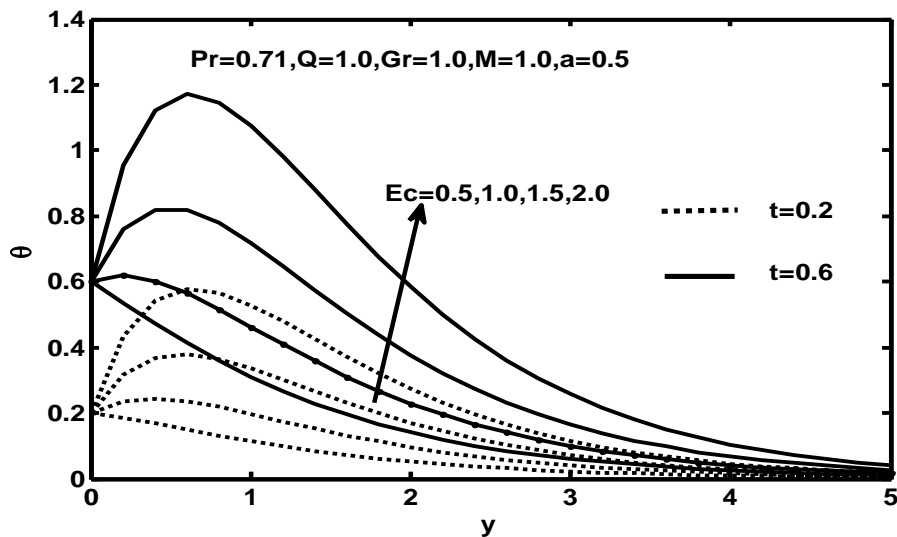


Figure (10): Temperature profile for different values of E

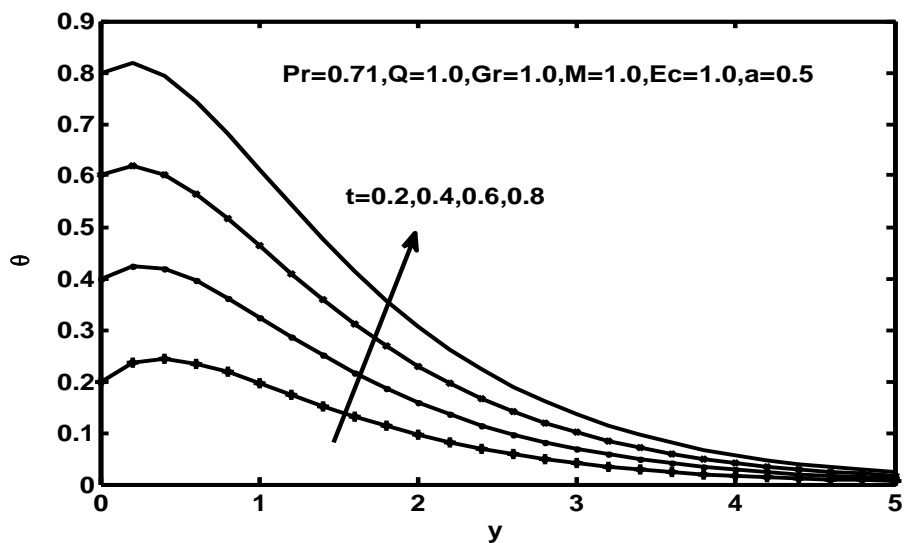


Figure (11): Temperature profile for different values of 't'

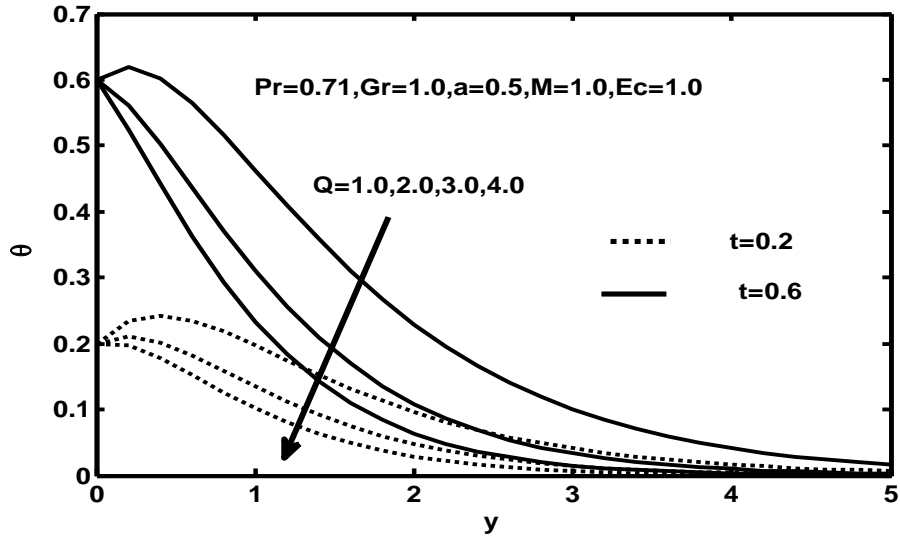


Figure (12): Temperature profile for different values of Q

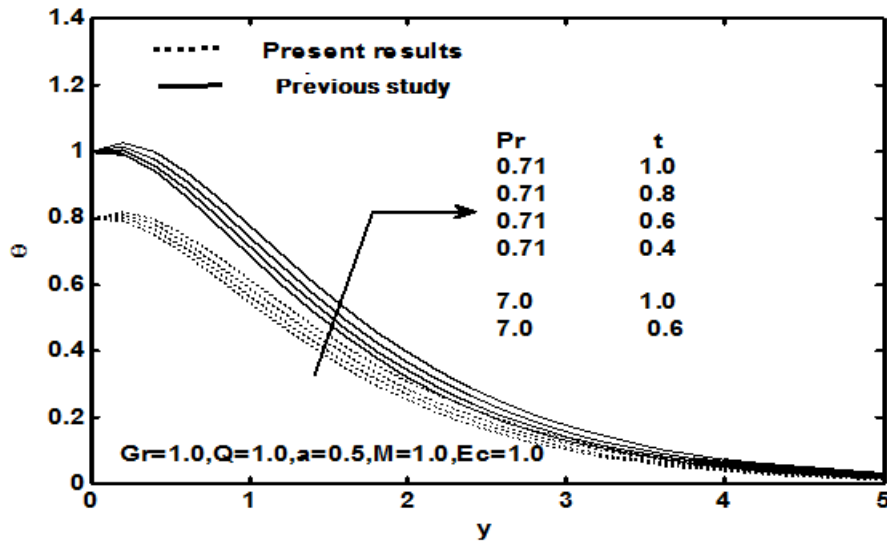


Figure (13): Temperature profile for different values of Pr and t

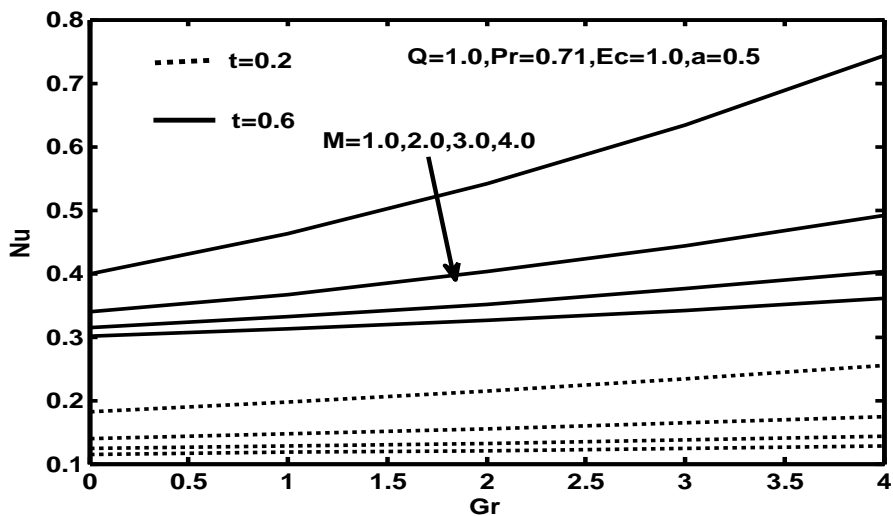


Figure (14): Nusselt number for different values of M versus Gr